FORMULÁRIO (26-3-2019)

- α – alfa
- β – beta
- χ – qui
- δ – delta
- ε – eps
- λ – lambda
- μ – miu
- ν – niu
- π – pi
- θ – theta
- ρ – rho
- τ – tau
- ω – omega
- Ω – OMEGA
- ξ – qsi

Refracção

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

$$\theta_{Crítico} = \mathrm{asin}(n_2 / n_1)$$

$$\theta_{Brewster} = \mathrm{atan}(n_2 / n_1)$$

Prismas

$$\delta \approx (n-1)\alpha$$

$$\delta = \theta_{i1} - \alpha + \mathrm{asin}\left[\sqrt{n^2 - \sin^2(\theta_{i1})}\sin(\alpha) - \sin(\theta_{i1})\cos(\alpha)\right]$$

$$\theta_{i1} = (\delta_m + \alpha)/2$$

$$n = \sin\left(\frac{\delta_m + \alpha}{2}\right) \Big/ \sin\left(\frac{\alpha}{2}\right)$$

Equações de Fresnel

$$r_{//} = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_i \cos\theta_t + n_t \cos\theta_i} \qquad t_{//} = \frac{2 n_i \cos\theta_i}{n_i \cos\theta_t + n_t \cos\theta_i}$$

$$r_{\perp} \equiv \left(\frac{E_{0r}}{E_{0i}}\right) = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t} \qquad t_{\perp} \equiv \left(\frac{E_{0t}}{E_{0i}}\right) = \frac{2 n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t}$$

$$R_{\perp} = r_{\perp}^2 \qquad R_{\parallel} = r_{\parallel}^2$$

$$T_{\perp} = \left(\frac{n_t \cos\theta_t}{n_i \cos\theta_i}\right) t_{\perp}^2 \qquad T_{\parallel} = \left(\frac{n_t \cos\theta_t}{n_i \cos\theta_i}\right) t_{\parallel}^2$$

$$z_0 = \frac{\pi W_0{}^2}{\lambda} \qquad \Re = \left(\frac{n_2 - n_1}{n_1 + n_2}\right)^2$$

$\tan \theta_B = n_t / n_i$

$\sin \theta_c = n_t / n_i$

Óptica Geométrica

$$\frac{n}{l} + \frac{n'}{l'} = K \qquad K \equiv \frac{n}{f} = \frac{n'}{f'} \qquad M_L = -M_T{}^2$$

$$m \equiv M_T \equiv \frac{h'}{h} = -\frac{n\, l'}{n'\, l} \qquad \frac{1}{l} + \frac{1}{l'} = \frac{1}{f}$$

$$K = K_1 + K_2 - \frac{d}{n_l} K_1 K_2$$

$$\delta = \frac{n}{n_l}\frac{dK_2}{K} \qquad (n - n_l - n')$$

$$\delta' = -\frac{n'}{n_l}\frac{dK_1}{K} = -\delta \frac{n'}{n}\frac{K_1}{K_2}$$

$$K_{L-espessa} = (n_l - n_{meio})\left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{d(n_l - n_{meio})}{n_l R_1 R_2}\right)$$

$$\delta = -\frac{f(n_l - 1)d}{n_l R_2}$$

$$\delta' = -\frac{f(n_l - 1)d}{n_l R_1} = \delta \frac{R_2}{R_1}$$

$$K = -\frac{2}{R} \qquad K = \frac{n' - n}{R}$$

$$\nabla^2 u(\mathbf{r}, t) - \frac{1}{c^2}\frac{\partial^2 u(\mathbf{r}, t)}{\partial t^2} = 0$$

$$(\nabla^2 + k^2) U(\mathbf{r}) = 0$$

$$u(\mathbf{r}, t) = U(\mathbf{r}) e^{-i2\pi\nu t} = U(\mathbf{r}) e^{-i\omega t}$$

$$k = \frac{2\pi\nu}{c} = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

$$U(\mathbf{r}) = A e^{-i\mathbf{k}.\mathbf{r}}$$

$$U(\mathbf{r}) = \frac{A}{r} e^{-ikr}$$

$$U(\mathbf{r}) = \frac{A}{z} e^{-ikz} e^{-ik\frac{x^2 + y^2}{2z}}$$

Ondas paraxiais

$$U(\mathbf{r}) = A(\mathbf{r}) e^{-ikz}$$

$$\nabla_T^2 A - i2k\frac{\partial A}{\partial z} = 0$$

Modo Gaussiano TEM_{00}

$$U(\rho, z) = A_0 \frac{W_0}{W(z)} e^{-\frac{\rho^2}{W^2(z)}} e^{-i\left[kz + \frac{k\rho^2}{2R(z)} - \zeta(z)\right]} \qquad \zeta = \mathrm{atan}\left(\frac{z}{z_0}\right)$$

$$W(z) = W_0\left[1 + \left(\frac{z}{z_0}\right)^2\right]^{1/2} \qquad R(z) = z\left[1 + \left(\frac{z_0}{z}\right)^2\right]$$

Modos de Hermite / Laguerre - Gauss

$$U_{l,m}(x, y, z) = A_{l,m}\frac{W_0}{W(z)} \mathbb{G}_l\left[\frac{\sqrt{2}x}{W(z)}\right] \mathbb{G}_m\left[\frac{\sqrt{2}y}{W(z)}\right] e^{\left[-ikz - ik\frac{x^2 + y^2}{2R(z)} + (l + m + 1)\zeta(z)\right]} \qquad \mathbb{G}_l(u) = \mathbb{H}_l(u) e^{-\frac{u^2}{2}}$$

$$U_{l,m}(\rho, \varphi, z) = A_{l,m}\frac{W_0}{W(z)}\left(\frac{\rho}{W(z)}\right)^{|l|} L_m^{|l|}\left[\frac{2\rho^2}{W^2(z)}\right] e^{-\frac{\rho^2}{W^2(z)}} e^{\left[-ikz - ik\frac{\rho^2}{2R(z)} - il\varphi + (l + 2m + 1)\zeta(z)\right]} \qquad \rho^2 = x^2 + y^2$$

Modos de Bessel

$$U(\mathbf{r}) = A(x, y) e^{-i\beta z}$$

$$\nabla_T^2 A + k_T^2 A = 0$$

$$A(x, y) = A_m J_m(k_T \rho) e^{-im\phi}, \qquad m = 0, \pm 1, \pm 2, \ldots$$

$$k_T^2 + \beta^2 = k^2$$

$$\nabla_T^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$$

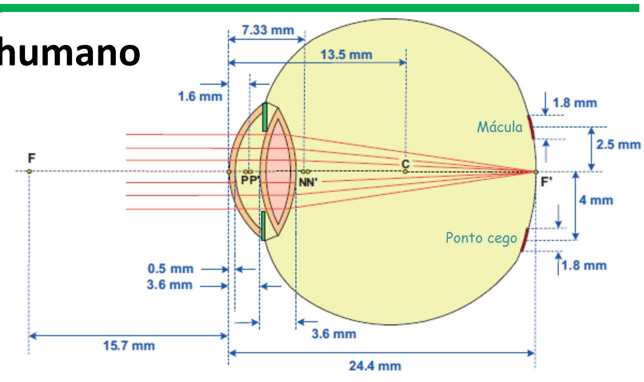
	NÃO ACOMODADO (NA)				ACOMODADO (A)			
SUPERFÍCIE	Z	R	n	K	Z	R	n	K
Córnea - 1ª superfície	0	7,7000	1,3760	48,83	0,0000	7,7000	1,3760	48,8300
Córnea - 2ª superfície	0,5	6,8000	1,3360	-5,88	0,5000	6,8000	1,3360	-5,8800
Cristalino - 1ª superfície	3,6	10,0000	1,4085	5	3,2000	5,3300	1,4260	9,3750
Cristalino - 2ª superfície	7,2	-6,0000	1,3360	8,33	7,2000	-5,3300	1,3360	9,3750

Posições relativas ao vértice da córnea (z=0), em mm

		CÓRNEA		CRISTALINO		OLHO COMPLETO		
GRANDEZA FÍSICA		NA	A	NA	A	NA	A	
Potência	K	43,053	43,053	19,11	33,06	58,636	70,57	K
Ponto Principal Objecto	Z(H)	-0,0496	-0,0496	5,678	5,145	1,348	1,772	Z(H)
Ponto Principal Imagem	Z(H')	-0,0506	-0,0506	5,807	5,225	1,602	2,086	Z(H')
Ponto Focal Objecto	Z(F)					-15,707	-12,397	Z(F)
Ponto Focal Imagem	Z(F')					24,387	21,016	Z(F')
Distância focal Objecto	f	23,227	23,227	69,908	40,416	17,055	14,619	f
Distância focal Imagem	f'	31,031	31,031	69,908	40,416	22,785	18,93	f'
Ponto Nodal Objecto	Z(N)					7,078	6,533	Z(N)
Ponto Nodal Imagem	Z(N')					7,332	6,847	Z(N')
Posição da Pupila de Entrada						3,045	2,667	
Posição da Pupila de Saída						3,664	3,211	

Distâncias focais e posições relativas ao vértice da córnea (z=0), em mm

Olho humano



Difracção

$$U(x_0,y_0) = \frac{1}{i\lambda}\iint_{\Sigma} U(x_1,y_1)\frac{e^{ikr_{01}}}{r_{01}}\cos\theta\, dx_1dy_1$$

$$U(x,y) = \frac{e^{ikz}}{i\lambda z}\iint_{-\infty}^{\infty} U(\xi,\eta)e^{-i\frac{k}{2z}\left[(x-\xi)^2+(y-\eta)^2\right]}d\xi d\eta$$

$$U(x,y) = \frac{e^{ikz}e^{-i\frac{k}{2z}(x^2+y^2)}}{i\lambda z}\iint_{-\infty}^{\infty} U(\xi,\eta)e^{-i\frac{2\pi}{\lambda z}(x\xi+y\eta)}d\xi d\eta = \frac{e^{ikz}e^{-i\frac{k}{2z}(x^2+y^2)}}{i\lambda z}\mathcal{F}\{U\}_{f_X=\frac{x}{\lambda z},\, f_Y=\frac{y}{\lambda z}}$$

Transformação de Fourier

$$\mathcal{F}\{g\} = G(f_X,f_Y) = \iint_{-\infty}^{\infty} g(x,y)e^{-i2\pi(f_Xx+f_Yy)}dxdy$$

$$\iint_{-\infty}^{\infty}|g(x,y)|^2dxdy = \iint_{-\infty}^{\infty}|G(f_X,f_Y)|^2df_Xdf_Y$$

$$\mathcal{F}^{-1}\{G\} = g(x,y) = \iint_{-\infty}^{\infty} G(f_X,f_Y)\,e^{+i2\pi(f_Xx+f_Yy)}df_Xdf_Y$$

$$g \star\star\, \delta = \iint_{-\infty}^{\infty} g(x,y)\,\delta(x-\xi,y-\eta)\,dxdy = g(\xi,\eta)$$

$$\mathcal{F}\mathcal{F}^{-1}\{g(x,y\} = \mathcal{F}^{-1}\mathcal{F}\{g(x,y\} = g(x,y)$$

$$\mathcal{F}\left\{\iint_{-\infty}^{\infty} g(\xi,\eta)\,h(x-\xi,y-\eta)\,d\xi d\eta\right\} = G(f_X,f_Y)H(f_X,f_Y)$$

$$\mathcal{F}\{\alpha g+\beta h\} = \alpha\mathcal{F}(g)+\beta\mathcal{F}(h) = \alpha G(f_X,f_Y)+\beta H(f_X,f_Y)$$

$$\mathcal{F}\{gh\} = G(f_X,f_Y)\star\star\, H(f_X,f_Y)$$

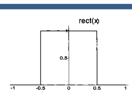
$$\mathcal{F}\{g(x-a,y-b)\} = G(f_X,f_Y)\,e^{-i2\pi(f_Xa+f_Yb)}$$

$$\mathcal{F}\left\{\iint_{-\infty}^{\infty} g(\xi,\eta)\,g^*(\xi-x,\eta-y)\,d\xi d\eta\right\} = \mathcal{F}\{g\otimes\otimes g\} = |G(f_X,f_Y)|^2$$

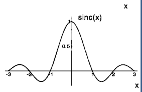
$$\mathcal{F}\{g(ax,by)\} = \frac{1}{|ab|}G\left(\frac{f_X}{a},\frac{f_Y}{b}\right)$$

J. Goodman, Introduction to Fourier Optics (3ª edição, 2005), Cap. 2

$$\text{rect}(x) = \begin{cases} 1 & |x| < \frac{1}{2} \\ \frac{1}{2} & |x| = \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

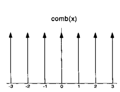


$$\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$



$$\text{circ}(\sqrt{x^2+y^2}) = \begin{cases} 1 & \sqrt{x^2+y^2} < 1 \\ \frac{1}{2} & \sqrt{x^2+y^2} = 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\text{comb}(x) = \sum_{n=-\infty}^{\infty}\delta(x-n)$$



$$\text{comb}\,\frac{x-x_0}{b} = |b|\sum_{n=-\infty}^{\infty}\delta(x-x_0-nb)$$

Function	Transform
$\exp[-\pi(a^2x^2+b^2y^2)]$	$\frac{1}{\lvert ab\rvert}\exp\left[-\pi\left(\frac{f_X^2}{a^2}+\frac{f_Y^2}{b^2}\right)\right]$
$\text{rect}(ax)\,\text{rect}(by)$	$\frac{1}{\lvert ab\rvert}\text{sinc}(f_X/a)\,\text{sinc}(f_Y/b)$
$\delta(ax,by)$	$\frac{1}{\lvert ab\rvert}$
$\exp[j\pi(ax+by)]$	$\delta(f_X-a/2,\,f_Y-b/2)$
$\text{comb}(ax)\,\text{comb}(by)$	$\frac{1}{\lvert ab\rvert}\text{comb}(f_X/a)\,\text{comb}(f_Y/b)$
$\exp[j\pi(a^2x^2+b^2y^2)]$	$\frac{j}{\lvert ab\rvert}\exp\left[-j\pi\left(\frac{f_X^2}{a^2}+\frac{f_Y^2}{b^2}\right)\right]$
$\exp[-(a\lvert x\rvert+b\lvert y\rvert)]$	$\frac{1}{\lvert ab\rvert}\frac{2}{1+(2\pi f_X/a)^2}\frac{2}{1+(2\pi f_Y/b)^2}$

$$\cos(2\pi f_0x) = \frac{e^{+i2\pi f_0x}+e^{-i2\pi f_0x}}{2}$$

$$\frac{1}{2}[\delta(f_X-f_0)+\delta(f_X+f_0)]$$

$$\sin(2\pi f_0x) = \frac{e^{+i2\pi f_0x}-e^{-i2\pi f_0x}}{2i}$$

$$\frac{1}{2i}[\delta(f_X-f_0)-\delta(f_X+f_0)]$$

Dieléctricos. Metais

$$\mathcal{P} = \epsilon_o\chi\mathcal{E}+\mathcal{P}_{\text{NL}},$$

$$\mathcal{P}_{\text{NL}} = 2\text{d}\mathcal{E}^2+4\chi^{(3)}\mathcal{E}^3+\cdots$$

$$k = \beta - i\frac{\alpha}{2} = k_0\sqrt{1+\chi'+i\chi''}$$

$$n - j\frac{1}{2}\frac{\alpha}{k_o} = \sqrt{\epsilon/\epsilon_o} = \sqrt{1+\chi'+j\chi''}$$

$$\chi(\nu) = \chi_0\frac{\nu_0^2}{\nu_0^2-\nu^2+j\nu\,\Delta\nu}$$

$$\chi'(\nu) = \chi_0\frac{\nu_0^2\,(\nu_0^2-\nu^2)}{(\nu_0^2-\nu^2)^2+(\nu\,\Delta\nu)^2}$$

$$\chi''(\nu) = -\chi_0\frac{\nu_0^2\,\nu\,\Delta\nu}{(\nu_0^2-\nu^2)^2+(\nu\,\Delta\nu)^2}$$

Absorção fraca

$$n \approx \sqrt{1+\chi'}$$

$$\alpha \approx -\frac{k_o}{n}\chi''.$$

$$\epsilon_e = \epsilon + \frac{\sigma}{j\omega}$$

$$n \approx \sqrt{\sigma/2\omega\epsilon_o}$$

$$\alpha \approx \sqrt{2\omega\mu_o\sigma}$$

$$\eta \approx (1+j)\sqrt{\omega\mu_o/2\sigma}$$

$$d_p = 1/\alpha = 1/\sqrt{2\omega\mu_o\sigma}$$

$$\varepsilon_e = \varepsilon_0\left(1-\frac{\omega_p^2}{\omega^2}\right)$$

$$k = \beta - j\tfrac{1}{2}\alpha = \omega\sqrt{\epsilon_e\mu_o}$$

$$\omega_p = \sqrt{\frac{\sigma}{\varepsilon_0\tau_c}} = \sqrt{\frac{Ne^2}{m\varepsilon_0}}$$

$$n - j\alpha/2k_o = \sqrt{\epsilon_e/\epsilon_o}$$